PHYC 581: High Energy Astrophysics
Fall 2014

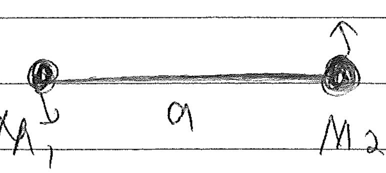
Lecture 18: 10/20/2014

Accretion from a Companion Star:

In many cases, isolated compact objects become visible because of accreting plasma from the surrounding medium. Supermassive black holes at the center of galaxies are examples of such a case. There are however situations where high-energy sources grow by absorbing matter from a companion in tight binary systems. Examples of such cases include nova and X-ray bursts.

We start our discussion of accretion in binary systems by considering the motion of a test particle in such systems. Consider two objects of mass M_1 and M_2 orbiting around each other. According to Kepler's third law:

$$4\pi^2 a^3 = G(M_1 + M_2) P_{orb}^3$$



Here P_{orb} is the period of orbital motion and a is the binary separation. Going to the frame that co-rotates with the objects, the forces acting on the test particle are:

$$\vec{F}_1 = \frac{-GM_1 m}{|r - \vec{r}_1|^3}(\vec{r} - \vec{r}_1), \quad \vec{F}_2 = \frac{-GM_2 m}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2), \quad \vec{F}_{cf} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

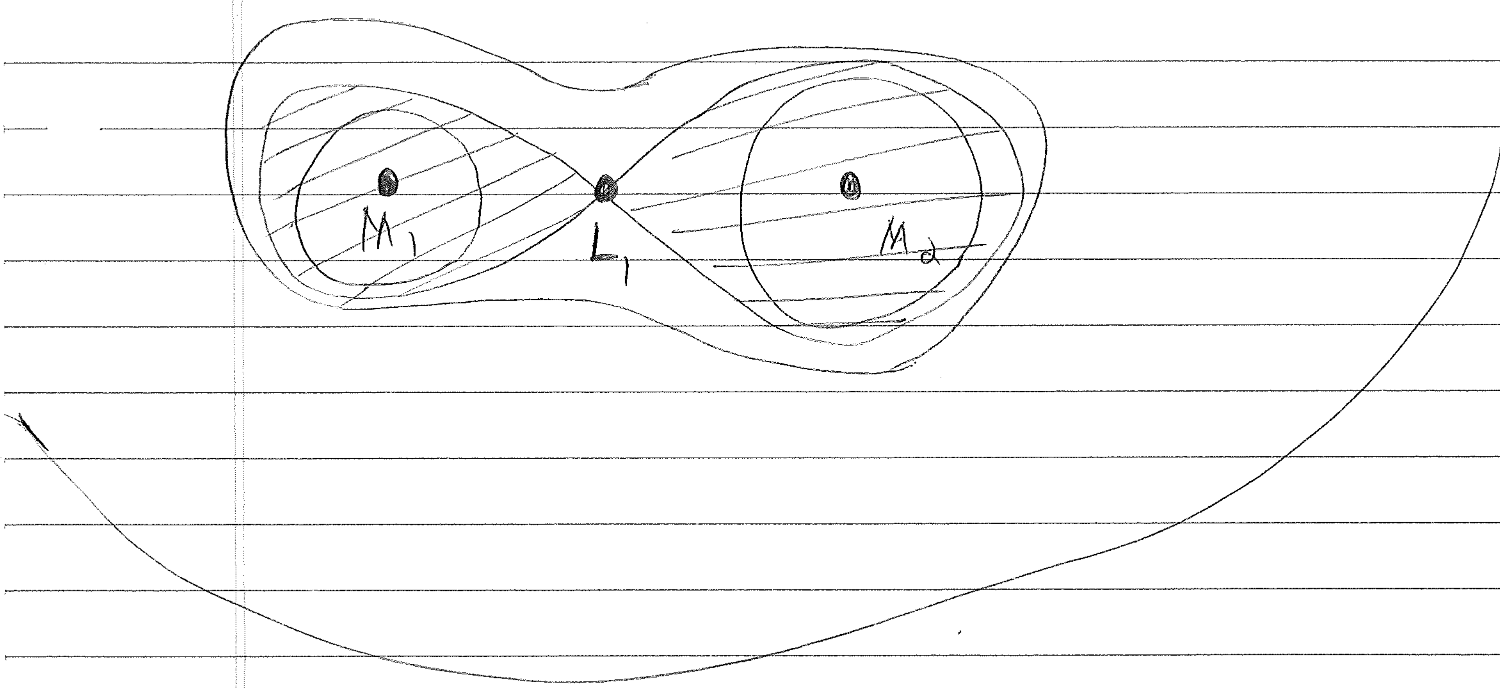
Here all the positions are relative to the center-of-mass of the binary system, and $\vec{\omega}$ is the angular velocity vector.

Note that $\vec{F}_{cf}$ (the centrifugal force) is a fictitious force (Coriolis force is neglected because it is proportional to the velocity). The total potential of the test particle in the co-rotating frame, called Roche potential, can be written as:

$$\Phi_R(\vec{r}) = -\frac{GM}{|\vec{r} - \vec{r}_1|} - \frac{GM_2}{|\vec{r} - \vec{r}_2|} - \frac{1}{2}|\vec{\omega}\times\vec{r}|^2$$

One can find useful information about the behavior of the test particle by drawing equipotential surfaces. For

$r \gg a$, the equipotential surfaces are circles whose centers are at the center-of-mass of the system. When $|\vec{r}-\vec{r}_1|$ is very small, equipotential surfaces are circles centered around the position of M_1. Similarly, when $|\vec{r}-\vec{r}_2|$ is very small, the equipotential surfaces are circles centered around the position of M_2. The situation can be summarized as follows.



It is seen that at some point the equipotential surface intersects itself. The intersection point L_1 is one of the 5 Lagrange points of the system. It represents an unstable equilibrium point along the direction that connects the center of M_1 and M_2. Therefore a test particle can slide across L_1 from M_1 (M_2) toward M_2 (M_1). The shaded regions in the figure represent two lobes that are called the Roche lobe of the two masses.

Accretion of mass in a binary system can now be understood in the content of Roche lobes. If one of the objects, say M_2, is big enough to fill its Roche lobe, then the mass can accrete from its outer layers to the other object (M_1). The average radius of the Roche lobe of M_2 is given by:

$$\frac{R_2}{a} \simeq 0.38 + 0.20 \log q \qquad 0.5 \leq q < 20$$

$$\frac{R_2}{a} \simeq 0.462 \left(\frac{q}{1+q}\right)^{\frac{1}{3}} \qquad 0 < q < 0.5 \qquad \left(q \equiv \frac{M_2}{M_1}\right)$$

To find the average radius of the Roche lobe of M_1 we need to change $q \to \frac{1}{q}$, hence:

$$\frac{R_1}{a} \simeq 0.38 - 0.20 \log q \qquad 0.05 < q < 2$$

$$\frac{R_1}{a} \simeq 0.462 \left(\frac{1}{1+q}\right)^{\frac{1}{3}} \qquad 2 \leq q$$

Also, the distance of the Lagrange point L_1 from the center of M_1 is given by:

$$\frac{b_1}{a} = 0.500 - 0.227 \log q$$

It is seen that L_1 is closer to the center of the lighter of the objects (as expected).

Once the mass accretes from M_2 to M_1, then q will change. As a result, R_1 and R_2 as well as b_1 and a will also change. The question that arises is whether the accretion will be sustained. Equivalently, whether M_2 will still be bigger than the average radius of its Roche lobe. To examine the situation, we assume the following:

(1) $M = M_1 + M_2 = \text{Const.}$

(2) $\vec{L} = \vec{L}_1 + \vec{L}_2 = \text{Const.}$

We therefore ignore processes like stellar winds that can transfer mass and angular momentum away from the system.

From the conservation of the total mass we have:

$M_1(1+q) = \text{Const.}$

Conservation of the total angular momentum results in:

$M_1 a_1^2 \omega + M_2 a_2^2 \omega = \text{Const.}$

Here a_1 and a_2 are the distances of M_1 and M_2 centers from the center-of-mass respectively, which are given by:

$a_1 = \frac{qa}{1+q}$, $a_2 = \frac{a}{1+q}$

Since $\omega = \frac{2\pi}{P_{orb}}$, we find:

$\frac{q}{1+q} \frac{M_1 a^2}{P_{orb}} = \text{Const.}$

Using Kepler's third law, and since $M_1 + M_2$ is constant, we have:

$a^2 \propto P_{orb}^{\frac{4}{3}} \Rightarrow P_{orb} \propto \frac{(1+q)^6}{q^3} \propto \frac{1}{M_1^3 M_2^3}$

Thus:

$$a \propto \frac{(1+q)^4}{q^3} \propto \frac{1}{M_1^2 M_2^2}$$

We conclude that "a" and P_{orb} are minimum when $q=1$. If $q>1$, reduction of q because of the accretion results in a decrease in "a" (the opposite occurs when $q<1$). Using the equation for R_2, we then see that:

$$R_2 = f(q)\, a \qquad f(q) = \begin{cases} 0.38 + 0.20 \log q & 0.5 \le q < 20 \\ 0.462 \left(\frac{q}{1+q}\right)^{1/3} & 0 < q < 0.5 \end{cases}$$

$$\frac{\Delta R_2}{R_2} = \left(\frac{f'}{f} + \frac{a'}{a}\right) \Delta q$$

Here f' and a' denote differentiation with respect to q.

It can be shown that $\left|\frac{a'}{a}\right| > \left|\frac{f'}{f}\right|$, and hence:

$$\frac{\Delta R_2}{R_2} \sim \frac{a'}{a} \Delta q = \frac{\Delta a}{a}$$

This implies that $\Delta a < 0$ results in $\Delta R_2 < 0$, while $\Delta a > 0$ leads to $\Delta R_2 > 0$. Having $\Delta R_2 < 0$ guarantees that accretion will be self-sustained. This requires that $\Delta a < 0$, due to accretion, which happens when $q > 1$.

We conclude that accretion will be sustained as long as $M_2 > M_1$, i.e., a compact object accreting from its big companion. In real systems, other effects can sustain accretion even if $q < 1$. For example, if the companion star is out of equilibrium, as would occur after mass loss, its outer envelope swells and fills its Roche lobe. This will further drive the binary system toward a more compact configuration.